

MATHEMATICAL GAMES

A new collection of short problems and the answers to some of "life's"

by Martin Gardner

Here is a collection of varied short problems, unrelated to one another. They will be answered in next month's department.

1. Enormously long chainlike molecules (long in relation to their breadth) have been discovered in living organisms. The question has arisen: Can such molecules have knotted forms? Max Delbrück of the California Institute of Technology, who received a Nobel prize last year, proposed the following idealized problem (in a paper that I shall identify next month):

Assume that a chain of atoms, its ends joined to form a closed space curve, consists of rigid, straight-line segments each one unit long. At every node where two such "links" meet, a 90-degree angle is formed. At each end of each link, therefore, the next link may have one of four different orientations. The entire closed chain could be traced along the edges of a cubical lattice, with the proviso that at each node the joined links form a right angle [see illustration below]. At no point is the chain allowed to touch or intersect itself; that is, two and only two links meet at every node.

What is the shortest chain of this type that is tied in a single overhand (trefoil)

knot? Next month I shall reproduce the shortest chain Delbrück has found. It has not been proved minimal; perhaps a reader will discover a shorter one. (I wish to thank John McKay for calling this problem to my attention.)

2. The old problem of expressing integers with four 4's (discussed in an earlier column reprinted as Chapter 5 of my *Numerology of Dr. Matrix*) has been given many variations. In an intriguing new variant proposed by Fitch Cheney one is allowed to use only pi and symbols for addition, subtraction, multiplication, division, square root and the "round-down function." In the last operation, indicated by brackets, one takes the greatest integer that is equal to or less than the value enclosed by the brackets. Parentheses also may be used, as in algebra, but no other symbols are allowed. Each symbol and pi may be repeated as often as necessary, but the desideratum is to use as few pi symbols as possible. For example, 1 can be written $[\sqrt{\pi}]$ and 3 even more simply as $[\pi]$.

The reader is invited to do his best to express the integers from 1 through 20 according to these rules, and to compare them next month with the best Cheney was able to achieve.

3. L. Vosburgh Lyons contributed a fiendish dissection problem to a magic magazine in 1969 [see top illustration on opposite page]. The polygon [at left in illustration] can be dissected into four

congruent polygons [at right]. Can the reader discover the only way in which the same polygon can be cut into five congruent polygons?

4. A full set of 32 chessmen is placed on a chessboard, one piece to a cell. A "move" consists in transferring a piece from the cell it is on to any empty cell. (This has nothing to do with chess moves.) Gilbert W. Kessler, a mathematics teacher in a Brooklyn high school, thought of the following unusual problem: How can you place the 32 pieces so that a maximum number of transfer moves are required to arrange the pieces in the correct starting position for a game of chess?

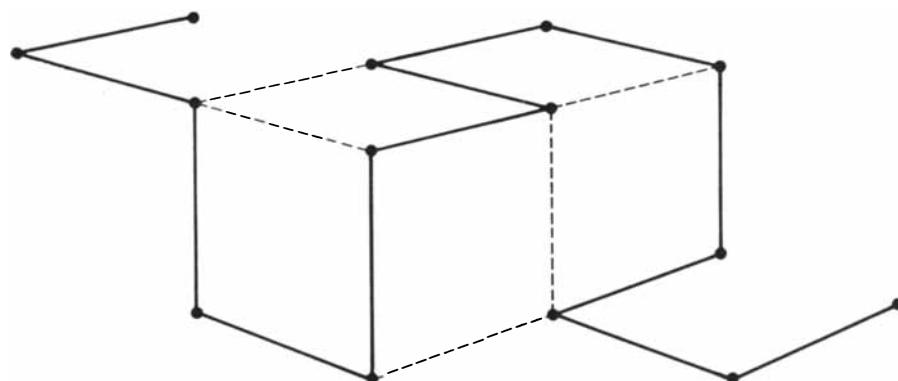
It is not specified which side of the board is the black side, but the playing sides must, as in regulation chess, be sides with a white square in the bottom right corner, and of course the queen must go on a square matching her own color. One is tempted at first to think that the maximum is 33 moves, but the problem is trickier than that.

5. A Texas oilman who was an amateur number theorist opened a new bank account by depositing a certain integral number of dollars, which we shall call x . His second deposit, y , also was an integral number of dollars. Thereafter each deposit was the sum of the two previous deposits. (In other words, his deposits formed a generalized Fibonacci series.) His 20th deposit was exactly a million dollars. What are the values of x and y , his first two deposits? (I am indebted to Leonard A. Monzert of West Newton, Mass., for sending me the problem of which this is a version.)

The problem reduces to a Diophantine equation that is somewhat tedious to solve, but a delightful shortcut using the golden ratio becomes available if I add that x and y are the two positive integers that begin the longest possible generalized Fibonacci chain ending in a term of 1,000,000.

6. A deck of 52 playing cards is shuffled and placed face down on the table. Then, one at a time, the cards are dealt face up from the top. If you were asked to bet in advance on the distance from the top of the first black ace to be dealt, what position (first, second, third, ...) would you pick so that if the game were repeated many times, you would maximize your chance in the long run of guessing correctly? In other words, what is the most probable position of the first black ace?

7. John Horton Conway, whose game "life" was last month's topic, defines a "quintomino" as a regular pentagon



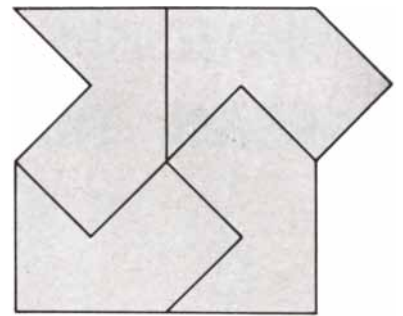
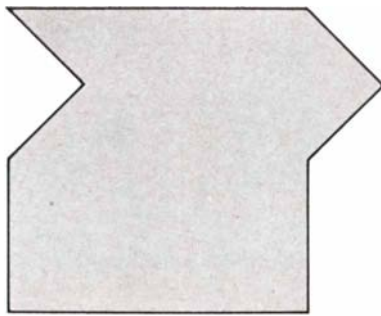
Example of a 13-link chain

whose edges (or triangular segments) are colored with five different colors, one color to an edge. Not counting rotations and reflections as being different, there are 12 distinct quintominoes. Letting 1, 2, 3, 4, 5 represent the five colors, the 12 quintominoes can be symbolized as follows:

- | | |
|----------|----------|
| A. 12345 | G. 13245 |
| B. 12354 | H. 13254 |
| C. 12435 | J. 13425 |
| D. 12453 | K. 13524 |
| E. 12534 | L. 14235 |
| F. 12543 | M. 14325 |

The numbers indicate the cyclic order of colors going either clockwise or counterclockwise around the pentagon [see illustration at left below]. In 1958 Conway asked himself if it was possible to color the edges of a regular dodecahedron [middle illustration below] in such a way that each of the 12 quintominoes would appear on one of the solid's 12 pentagonal faces. He found that it was indeed possible. Can readers find a way to do it?

Those who like to make mechanical puzzles can construct a cardboard model of a dodecahedron with small magnets glued to the inside of each face. The quintominoes can be cut from metal and colored on both sides (identical colors opposite each other) so that any piece can be "reflected" by turning it over. The magnets, of course, serve to hold the quintominoes on the faces of the solid while one works on the puzzle. The problem is to place the 12 pieces in such



L. Vosburgh Lyons' dissection problem

a way that the colors match across every edge.

Without such a model, the Schlegel diagram of a dodecahedron [illustration at right below] can be used. This is simply the distorted skeleton of the solid, with its back face stretched to become the figure's outside border. The edges are to be labeled (or colored) so that each pentagon (including the one delineated by the pentagonal perimeter) is a different quintomino.

8. Letters in the sentence "Roses are red, violets are blue" are scrambled by the following procedure. The words are written one below the other and flush at the left:

ROSES
ARE
RED
VIOLETS
ARE
BLUE

The columns are taken from left to

right and their letters from the top down, skipping all blank spaces, to produce this ordering:

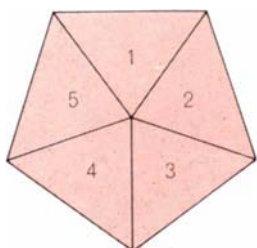
RARVABOREIRLSEDOEUELESETS.

The task is to find the line of poetry that, when scrambled by this procedure, becomes

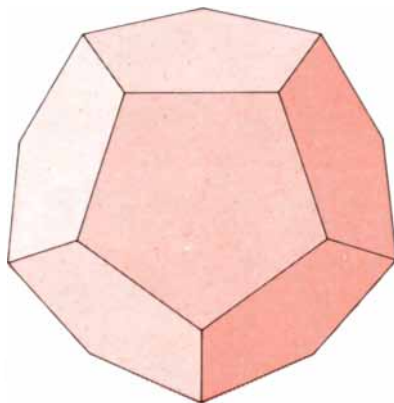
TINFLABTBULAHRSORIOOESAWEIKOANARG-EKRDYEASTE.

Walter Penney of Greenbelt, Md., contributed this novel word problem to the February 1970 issue of *Word Ways: The Journal of Recreational Linguistics*. That lively quarterly, now in its third year, is currently being published privately by A. Ross Eckler, Spring Valley Road, Morristown, N.J. 07960. A subscription is \$7 per year.

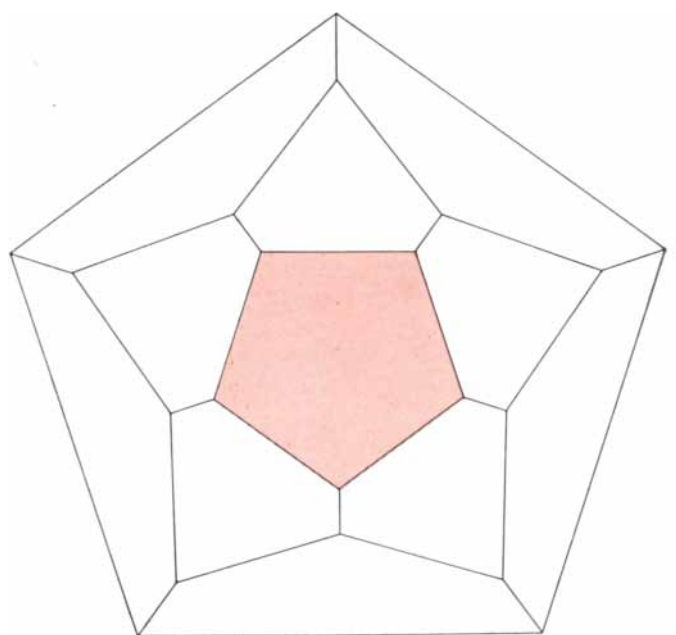
9. A secretary, eager to try out a new typewriter, thought of a sentence shorter than one typed line, set the controls for the two margins and then, starting at the



The A quintomino



The dodecahedron



Schlegel diagram of dodecahedron

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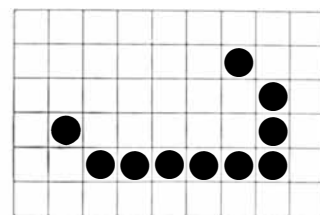
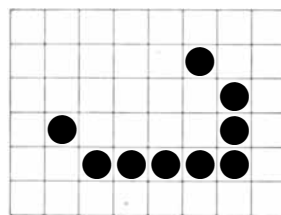
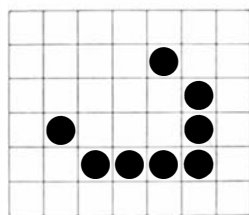
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Lightweight (left), middleweight (center) and heavyweight (right) spaceships

left and near the top of a sheet of paper, proceeded to type the sentence repeatedly. She typed the sentence exactly the same way each time, with a period at the end followed by the usual two spaces. She did not, however, hyphenate any words at the end of a line: when she saw that the next word (including whatever punctuation marks may have followed it) would not fit the remaining space on a line, she shifted to the next line. Each line, therefore, started flush at the left with a word of her sentence. She finished the page after typing 50 single-spaced lines.

Without experimenting on a typewriter, answer this question: Is there sure to be at least one perfectly straight column of blank spaces on the sheet, between the margins, running all the way from top to bottom? (T. Robert Scott originated this problem, which was sent to me by his friend W. Lloyd Milligan of Columbia, S.C.)

10. A: "What are the ages, in years only, of your three children?"

B: "The product of their ages is 36."

A: "Not enough information."

B: "The sum of their ages equals your house number."

A: "Still not enough information."

B: "My oldest child—and he's at least a year older than either of the others—has a wart on his left thumb."

A: "That's enough, thank you. Their ages are..."

Complete A's sentence. (Mel Stover of Winnipeg was the first of several readers to send in this problem, the origin of which I do not know.)

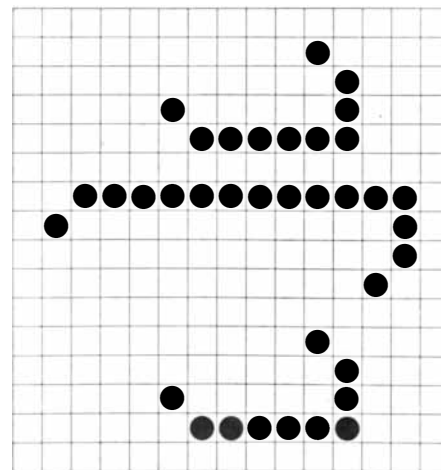
Last month's exercises, involving John Horton Conway's game "life," are answered as follows:

The Latin cross dies on the fifth move. The swastika turns into traffic lights (four blinkers) on the third move. The letter *H* dies on the sixth move. The next three figures are flip-flops: as Conway writes, "The toad pants, the clock ticks and the beacon flashes, with period 2 in every case." The pinwheel's interior rotates 90 degrees clockwise on each move, the rest of the pattern remaining stable. Periodic

figures of this kind, in which a fixed outer border is required to move the interior, Conway calls "billiard-table configurations" to distinguish them from "naturally periodic" figures such as the toad, clock and beacon.

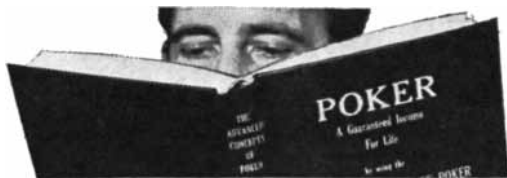
The three known spaceships (in addition to the glider, or "featherweight spaceship," given last month) are shown in the illustration above. All three travel horizontally to the right with half the speed of light. As they move they throw off sparks that vanish immediately as the ships continue on their way. Unescorted spaceships cannot have bodies longer than six counters without giving birth to objects that later block their motion. Conway has discovered, however, that longer spaceships, which he calls "overweight" ones, can be escorted by two or more smaller ships that prevent the formation of blocking counters. The illustration below shows the longest spaceship that can be escorted by as few as two smaller ships; still longer ships require a flotilla of more than two companions. A spaceship with a body of 100 counters, Conway finds, can be escorted safely by a flotilla of 33 smaller ships.

I shall report later on any success readers have in finding new spaceships or oscillators or in meeting Conway's challenge to settle his conjecture about the game.



Overweight spaceship with two escorts

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